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# List Metric Detection of Coded FH/MFSK in a Tone Jamming Environment

P. J. CREPEAU

Communication Systems Engineering Branch Information Technology Division

June 6, 1984



**NAVAL RESEARCH LABORATORY** Washington, D.C.

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6. NAME OF PERFORMING ORGANIZATION	66 OFFICE SYMBOL  If applicable	78 NAME OF MONITORING ORGANIZATION				
Naval Research Laboratory	Code 7523	Naval Air Syst	ems Comman	d		
6c ADDRESS City State and 71P Code	-	76 ADDRESS Cits State and /IP Code				
Washington, DC 20375		Washington, DC 20361				
Se NAME OF FUNDING SPONSORING ORGANIZATION	8b OFFICE SYMBOL  If applicable:	9 PROCUREMENT	INSTRUMENT ID	ENTIFICATION NUMBER		
Sc. ADDRESS, City State and ZIP Code:	ACCRESS II to National APP Control		10 SOURCE OF FUNDING NOS			
		PROGRAM	PROJECT	TASK	TINU NROW	
		ELEMENT NO	NO	NO I	NO	
11 ToTUE Include Sequents Classification: (See page ii)	<del></del>	61153N14 A34R310A		058A	DN280-076	
12 PERSONAL AUTHORIS: Crepeau, P.J.		<del></del>		<u> </u>	·	
13a. TYPE OF REPORT 13b TIME		14 DATE OF REPORT, Yr. Mo. Day, 15 PAGE COUNT June 6, 1984 26				
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19 ABSTRACT (Continue on reverse if necessary as	1		<del></del>			
The performance of coded FH/MFSK with list metric detection is presented for two severe jamming channels, the single-tone and multiple-tone jamming channels. It is shown that list detection is a highly efficient soft decision quantization scheme for tone jamming provided that channel state information in the form of list position probabilities is available. Results indicate an improvement in Eb/No performance of greater than 10 dB over hard decision performance when large alphabets and large list sizes are used. Optimal jamming fractions are derived, and in all cases the optimal jamming is less than full-band.						
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P. J. Crepeau	(202) 767-3	397	Code 7523			

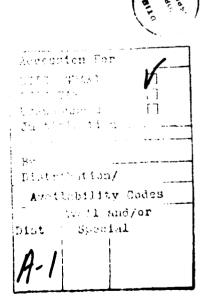
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#### LIST METRIC DETECTION OF CODED FH/MFSK IN A TONE JAMMING ENVIRONMENT

### INTRODUCTION

For uncoded frequency-hopped multiple frequency shift keying (FH/MFSK), Houston [1] has shown that the worst-case interference is intelligent tone jamming. Against this sophisticated form of jamming the performance of uncoded FH/MFSK systems can suffer degradations of greater than 50 dB. The importance of this can be seen by considering the maximum jamming-to-signal power ratio J/S which can be tolerated at a communication receiver. This is called the jamming margin and it is usually taken as a figure of merit for spread spectrum receivers. The jamming margin may be written as

$$\frac{J}{S} = \frac{W/R}{E_b/N_0} , \qquad (1)$$

where

J is the total jamming power,

S is the signal power.

W is the total spread (or hopped) bandwidth,

R is the information data rate,

E, is the energy per bit,

and  $N_o$  is the noise power spectral density.

Whereas frequency hopping can provide a processing gain W/R of greater than 50 dB, this advantage will be lost to the jamming threat for which  $\rm E_b/N_o$  can be of the order of 50 dB.

Manuscript approved March 30, 1984.

Much of the loss of the uncoded system to sophisticated tone jamming may be recovered by the use of error control coding [2]. Of critical importance is the metric information which is provided by the demodulator to the decoder. A pure soft-decision (analog energy) metric is not effective when the communication receiver does not have side-information concerning the state of the jamming signal. This is true because the receiver is vulnerable to a jamming strategy where high jamming power can be concentrated on a small number of symbols in a coded transmission sequence and lead to a large number of decoding errors.

An alternative soft-decision detection (quantization) scheme called list detection has been discussed in several previous papers and reports [3-6]. In this technique, demodulator energy outputs are ranked in magnitude from the highest to the lowest, and metrics are assigned according to position in the ranking. The process of rank-ordering is equivalent to partitioning the M-dimensional observation space of demodulator outputs into regions corresponding to different ordered lists. In this way, a discrete memoryless channel is created with more outputs than inputs as seen by the encoder/decoder. (The special case where the size of the list is equal to one reduces to hard decision quantization).

List detection has been considered previously for the additive white Gaussian noise channel where performance was found to be inferior to quantized energy metric detection. More recently list detection has been considered for fading and intelligent jamming environments [5,6] and the

results have shown large performance improvements for tone jamming channels. In this report we consolidate and extend the tone jamming results of [5,6].

Fundamental definitions and derivations are provided in [5] which is a predecessor to the present report. The key result in [5] which is used here is the expression for the cutoff rate parameter  $R_0$  for the list-of-L detection metric. This is given by

$$R_0 = \log_2 M - 2 \log_2 \left[ \sum_{k=1}^L \sqrt{q_k} + \sqrt{(M-L) q_0} \right],$$
 (2)

where  $q_k$  is the probability of the sent signal ranking in  $k^{th}$  position and  $q_0$  is the probability that the sent signal will rank lower than  $L^{th}$ . In order to achieve this value of  $R_0$  the receiver must have channel-state-information, that is, knowledge of the ranking probabilities  $\{q_k\}$ . The associated metrics for this situation are given by

$$N_{k} = \begin{cases} \log q_{k} ; & k = 1, 2, ..., L \\ \log \frac{q_{0}}{M-L} ; & k = L+1, L+2, ..., M \end{cases}$$
(3)

where the logarithm may be taken to any convenient base.

### FH/MFSK SIGNALING FORMAT AND TONE JAMMING

The FH/MFSK signaling format is shown in Figure 1. A total hopping bandwidth W is divided into b subbands with each tone symbol being transmitted on a different frequency hop. Within a hopping subband, one of M tones carrying k =  $\log_2$  M bits is sent with signal power S. Candidate tones are orthogonally spaced with a frequency spacing  $\Delta = \frac{1}{T_S} = R_S$  where  $T_S$  is the symbol duration and  $R_S$  is the symbol rate.

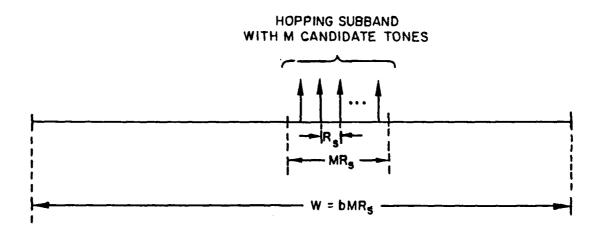


Fig. 1 - FH/MFSK signaling format

We consider two tone jamming models to be used against the FH/MFSK signaling format. In the first, jamming tones of equal amplitude are assigned randomly to subbands with a maximum of one tone per subband. In the second, tones of equal amplitude are assigned randomly to candidate signal tone slots with a maximum of one per tone slot, allowing for at most M jamming tones per subband. In both cases, it is assumed (perhaps unrealistically), that the

jammer has perfect knowledge of the communicator's signal power level, timing, and location of the frequency slots. All that is unknown to the jammer is the spread spectrum hopping code and the phase of the received signal tone.

#### SINGLE-TONE JAMMING

The single-tone jamming model was considered by Houston [1], but only for uncoded communication. In this report we extend the analysis to coded performance with list detection.

For the single-tone jamming model, the only two output ranking possibilities are that the correct signal scores first or second on the list. For the correct signal to rank first, the subband with the signal tone must either be free of a jamming tone or have a jamming tone which coincides with the signal tone. For the correct signal to rank second, there must be a jamming tone in the transmission subband at a non-transmitted frequency. Furthermore, the jamming tone must have more power than the signal tone.

Let the number of subbands with a jamming tone be N. For total jamming power J the jamming power per tone is J/N, and the signal-to-jamming tone power ratio is

$$\alpha = \frac{S}{J/N} \qquad . \tag{4}$$

For list-of-1 (hard-decision) detection the  ${\rm R}_{_{\mbox{\scriptsize O}}}$  expression (2) reduces to

$$R_0 = \log_2 M - 2 \log_2 \left[ \sqrt{q_1} + \sqrt{(M-1)} q_2 \right].$$
 (5)

To minimize  $R_0$ , the optimal jamming strategy is to minimize  $q_1$  and, consequently, to maximize  $q_2 = 1 - q_1$ . This can be done by choosing  $\alpha$  equal to 1-, a number slightly less than 1 (but taken equal to 1 in the ensuing calculations). With N subbands jammed the symbol error probability becomes

$$P_{s} = q_{2} = \frac{N}{b} \frac{M-1}{M}$$
 (6)

Since N = J/S and the number of subbands is  $b = W/MR_S$  (see Figure 1), it follows that

$$q_2 = \frac{M-1}{E_S/N_O}, \frac{E_S}{N_O} \ge M^*,$$
 (7)

where  $E_s = S/R_s$  is the symbol energy, and  $N_o = J/W$  is the equivalent noise power spectral density.

<sup>\*</sup> For  $E_S/N_0 < M$ , all subbands have jamming tones and  $\alpha = S/J/b < 1$ . Then  $q_2 = (M-1)/M$ ,  $q_1 = 1/M$ , and from (5),  $R_0 = 0$ .

Similarly, the probability of the correct signal ranking first is

$$q_1 = 1 - q_2 = 1 - \frac{M-1}{E_s/N_0}, \frac{E_s}{N_0} \ge M.$$
 (8)

With these values of  $q_1$  and  $q_2$ ,  $R_0$  may be found from (5).

With the R $_0$  result, it is useful to plot  $E_b/N_0$  versus  $\pm$  code rate R when the code rate is equal to the cutoff rate normalized to channel bits instead of channel symbols. Since there are  $\log_2 M$  channel symbol, the code rate (at cutoff) is defined as

$$R = \frac{R_0}{\log_2 M} \tag{9}$$

and it follows that

$$\frac{E_b}{N_o} = \frac{1}{R \log_2 M} \frac{E_s}{N_o}$$

$$= \frac{1}{R_o} \frac{E_s}{N_o} . \tag{10}$$

Curves of  $E_b/N_o$  versus R for hard-decision detection on the singletone channel for M = 4, 8, 16, and 32 are shown as the L=1 dashed curves in Figures 3-6 (shown at end of report). The increase in  $E_b/N_o$  at very low code rate is due to combining losses associated with noncoherent systems with redundancy.

For list-of-2 detection the cutoff rate parameter is given by

$$R_0 = \log_2 M - 2 \log_2 \left[ \sqrt{q_1} + \sqrt{q_2} \right]$$
 (11)

Here the optimal jamming strategy is to attempt to force  $q_2 = q_1 = 1/2$  so that

$$R_0 = \log_2 M - 1$$
 (12)

The condition  $q_1=q_2=1/2$  occurs when M/2(M-1) of the subbands possess a jamming tone. Thus the worst-case jammer will try to fill as many of the subbands up to M/2(M-1) as possible by using a jamming tone power slightly greater than the signal power ( $\alpha=1$ ). This is true for the range  $E_s/N_0>2(M-1)$ .

For  $E_s/N_0 < 2(M-1)$ , and  $\alpha = l_-$ ,  $q_2$  exceeds  $q_1$  and the communicator would be able to assign a higher metric (log  $q_2$ ) to the second ranking output than to the first ranking output (log  $q_1$ ). To counter this list-metric strategy, the jamming should attempt to maintain the condition  $q_1 = q_2 = 1/2$  by increasing the jamming tone power but not jamming more than M/2(M-1) of the subbands\*. Then we have  $\alpha < l_-$  and J/N > S in the region  $E_s/N_0 < 2(M-1)$ .

<sup>\*</sup> This point was not recognized in [5]. There the jammer was assumed to jam more than M/2(M-1) subbands using  $\alpha=1$  for  $E_S/N_0<2(M-1)$ . This is not optimal, however, since  $q_2>q_1$  and  $R_0$  increases from its minimum value of  $\log_2 M-1$ .

The curves of  $E_b/N_o$  vs R for list-of-2 detection for the single-tone jamming channel are shown in Figures 3-6\* as dashed lines labeled L=2. In each case  $E_b/N_o$  decreases to some value corresponding to a code rate R =  $(\log_2 M-1)/\log_2 M$  and then drops vertically. This occurs because the jamming strategy minimizes  $R_o$  at a value  $\log_2 M$  -1 and maintains it for all  $E_b/N_o$  values below the critical value

$$\frac{E_b}{N_0} = \frac{1}{R_0} \frac{E_s}{N_0} = \frac{2(M-1)}{\log_2 M - 1} . \tag{13}$$

The improvement for list detection from L=1 to L=2 for the single tone channel is large. In the next section, a more realistic tone jamming channel, the multiple-tone jamming channel, is considered.

#### MULTIPLE-TONE JAMMING

In the multiple-tone jamming channel [2] the jamming tone power levels are assumed to be all equal and randomly distributed with a maximum of one per slot over the total number of candidate tone slots. Referring to Figure 1, it is seen that there are M slots per subband and b subbands, so the total number of tone slots is bM. For N jamming tones the fraction  $\rho$  of slots jammed is given by

$$\rho = \frac{N}{bM} . ag{14}$$

<sup>\*</sup> List metric detection results are not shown for binary (M=2) signaling because list detection reduces to hard-decision detection in this case. That is, for M=2, the L=1 and L=2 results are identical because it is only the metric differences that are used by the decoder.

Since N =  $\alpha$ J/S and W = bMR<sub>s</sub> it follows from (14) that  $\rho$  and  $\alpha$  are related by

$$\rho = \frac{\alpha \frac{J}{S}}{M}$$

$$= \frac{\alpha}{E_S/N_O} , \qquad (15)$$

where

$$E_s = S/R_s$$
 ,

and

$$N_0 = \frac{J}{W} = \frac{J}{bMR_S}$$

For the multiple-tone jamming channel, there are four jamming possibilities which can exist within the transmitted subband. They are given as follows:

- (a) No jamming tones in subband,
- (b) Jamming tones in non-transmitted slots only,
- (c) Jamming tone in transmitted slot only,
- (d) Jamming tones in transmitted and non-transmitted slots.

In case (d), further complications arise depending upon the phase relationship between the signal and jamming tones in the transmitted slot [2]. This phase relationship is depicted in Figure 2. The signal tone amplitude is  $\sqrt{2S}$  and the jammer tone has amplitude  $\sqrt{\frac{2S}{\alpha}}$  and phase  $\Theta$  with respect to the

signal tone. The resultant amplitude is  $\sqrt{2S'}$  where

$$S' = S \left(1 + \frac{2}{\sqrt{\alpha}} \cos \Theta + \frac{1}{\alpha}\right) . \tag{16}$$

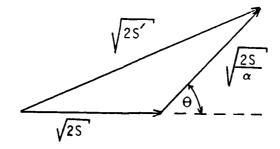


Fig. 2 - Phasor diagram for tone jammed signal

It is convenient to use an indicator random variable to denote whether or not a slot is jammed. Let  $T_x = 1$  indicate that the slot of a transmitted signal x is jammed ( $T_x = 0$  otherwise). Also let  $T_{\hat{x}} = 1$  indicate that a non-transmitted slot  $\hat{x}$  contains a jamming tone ( $T_{\hat{x}} = 0$  otherwise).

The energy from the MFSK matched filter for the transmitted tone slot is

$$\mathcal{E}_{x}^{2} = \begin{cases} E_{s} & , T_{x} = 0 \\ \left(1 + 2\sqrt{\alpha} \cos \theta + \alpha\right) \frac{E_{s}}{\alpha} & , T_{x} = 1 \end{cases} , \tag{17}$$

and for a non-transmitted tone slot the energy is

$$\mathcal{E}_{\hat{\chi}}^{2} = \begin{cases} 0 & , & T_{\hat{\chi}} = 0 \\ \frac{E_{S}}{\alpha} & , & T_{\hat{\chi}} = 1. \end{cases}$$
 (18)

The  $R_0$  expression for list metric detection is given in (2), and for the multiple-tone channel the ranking probabilities are given by

$$q_{k} = \sum_{n=0}^{M-1} {\binom{M-1}{n}} \rho^{n} \left(1-\rho\right)^{M-1-n} \left[ (1-\rho)q_{k}(n,0) + \rho q_{k}(n,1) \right], \quad (19)$$

where n is the number of jamming tones in non-transmitted tone slots of the transmission subband and

$$q_k(n,T_x) = P \{ signal tone x ranks k^{th} | n, T_x \}$$
.

If the signal tone is not jammed  $(T_x = 0)$ , then the signal places first on the list for  $\alpha > 1$ , and  $(n+1)^{th}$  for  $\alpha < 1$ . That is (using Kronecker delta notation),

$$q_{k}(n,o) = \begin{cases} S_{k}, & n+1 \\ S_{k}, & 1 \end{cases}, \quad \alpha < 1$$

$$(20)$$

If the signal tone is jammed ( $T_x = 1$ ), then the conditional ranking probabilities depend upon the relative phase between the signal and jamming tones. In this analysis a uniform distribution  $(0,2\pi)$  of relative phase between signal and jamming tones is assumed. For  $\alpha>4$ , the sent signal always places first on the list, but for  $\alpha<4$  it ranks first or  $(n+1)^{th}$  according to

$$q_{k}(n,1) = \begin{cases} \frac{\pi - \theta_{c}}{\pi} \delta_{k,n+1} + \frac{\theta}{\pi} \delta_{k,1}, & \alpha < 4 \\ \delta_{k,1}, & \alpha > 4 \end{cases}$$
(21)

where

$$\theta_{\rm c} = \cos^{-1}\left(\frac{\sqrt{\alpha}}{2}\right)$$
.

Thus when  $\ll >4$ , or when  $\ll <4$  and  $|\mathfrak{b}|<\mathfrak{h}_{\mathbb{C}}$ , the sent tone will rank first, otherwise it ranks  $(n+1)^{th}$ .

 $R_0$  is minimized by letting the jammer select  $\propto$  (equivalently,  $\rho$ ) for each value of  $E_s/N_0$ . The results of this minimization are plotted for various list sizes in Figures 3-6 in the normalized graphs of  $E_b/N_0$  versus code rate at cutoff. Also in Figure 7, the worst-case  $\rho$  is plotted versus  $E_s/N_0$  for the case of L=M for M = 4, 8, 16, and 32.

#### COMMENTS AND CONCLUSIONS

It has been shown that list detection is a robust soft-decision detection technique for coded FH/MFSK when used against tone jamming. For the single-tone channel the jammer is forced into a situation where the worst-case jamming is to make it equiprobable that the signal ranks first or second on the list. Then  $R_0$  is reduced by only 1 bit per channel use to a value of  $\log_2 M$  -1. To be more effective against a list metric, the jammer may adopt a multiple-tone strategy in an attempt to force a more nearly equiprobable ranking distribution and hence minimize  $R_0$ . In turn, the communicator may resort to larger list sizes in order to regain a large performance improvement.

The optimal strategy of the multiple-tone jammer is not full band ( $\rho$ =1) as in the case of an optimal noise-jammer [6]. This is true because if the jammer were to jam all slots, then the signal would rank first or last with about equal probability (assuming  $E_s/N_o$  and  $\propto$  small) and  $R_o$  would become  $\log_2 M$  -1, as in the single-tone, list-of-2 case.

Although previous studies [5,6] have shown list detection to be only moderately effective in Rayleigh fading and worst case partial band noise environments, the improvement is far greater against tone jamming. Improvement is greatest for large alphabets and large list sizes, and for M=32 with rate 1/2 coding and large list sizes (L=16 or 32), a reduction of  $E_b/N_0$  by 14 dB over hard decision performance is shown.

The major obstacle to implementation of a list metric detector is the difficulty in obtaining channel state information, that is, the ranking distribution  $\{q_k\}$ . One important case where channel state information may be readily available is the multiple-tone friendly-user multiple-access networking environment. Here the number of users and their power levels would serve to provide the needed channel state information.

It is recommended that future studies of the list metric for tone jamming channels include in the analysis the additive white Gaussian background noise. To date, the white Gaussian noise power has been considered as negligible compared to the jamming tone power, but in practical situations this may not be the case.

#### ACKNOWLEDGMENT

The author wishes to acknowledge the assistance received from Prof.

J. K. Omura and M. A. Creighton of UCLA in this study. Indeed, the bulk of the ideas presented relating to multiple-tone jamming channels stem from their original investigations. In addition, D. L. Tate of NRL was extremely helpful in performing the investigation of single-tone jamming channels.

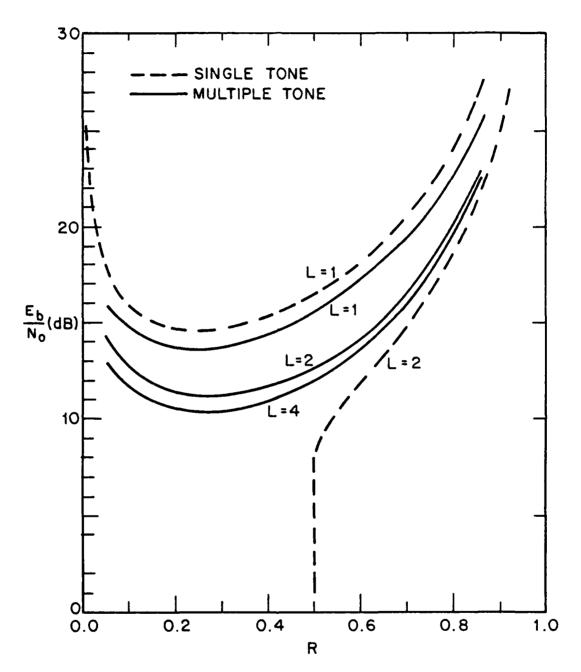


FIG. 3 -  $\frac{E_b}{N_o}$  vs Code Rate for Tone Channels (4FSK)

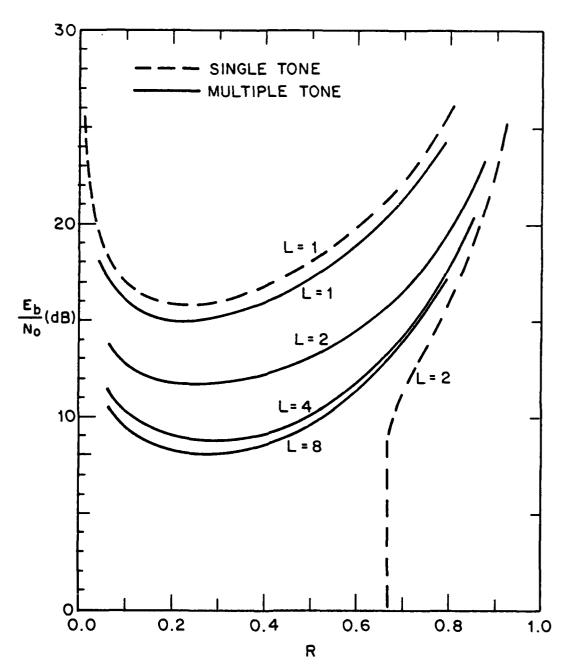


FIG. 4 -  $\frac{E_b}{N_o}$  vs Code Rate for Tone Channels (8FSK)

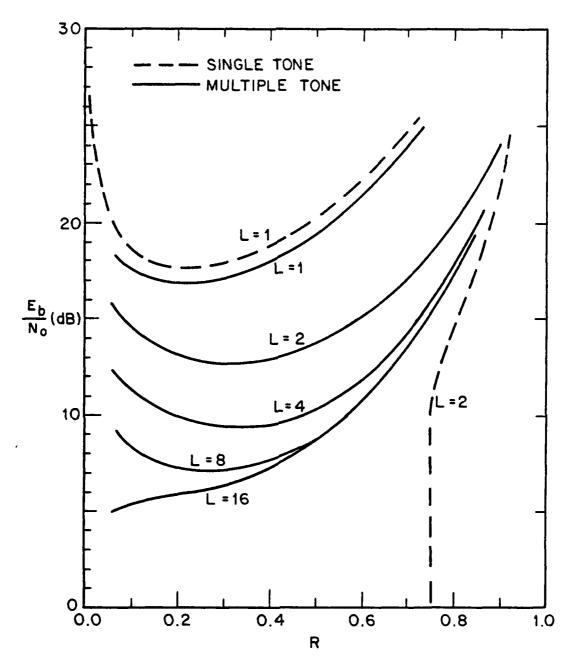


FIG. 5 -  $\frac{E_b}{N_o}$  vs Code Rate for Tone Channels (16FSK)

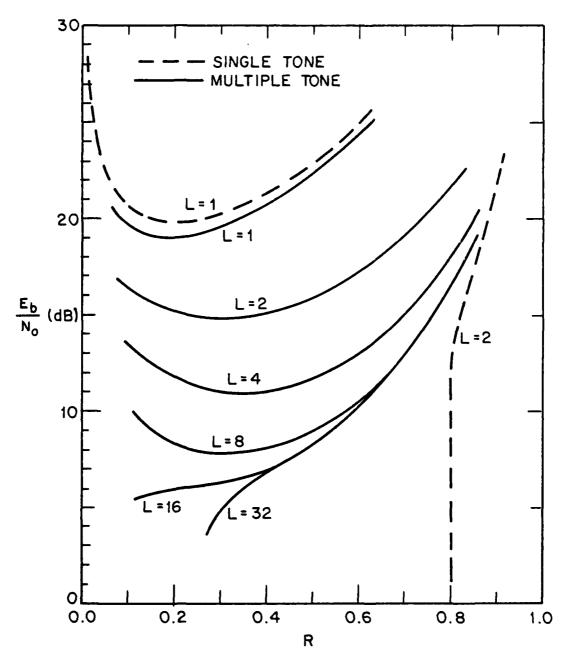


FIG. 6 -  $\frac{E_b}{N_o}$  vs Code Rate for Tone Channels (32FSK)

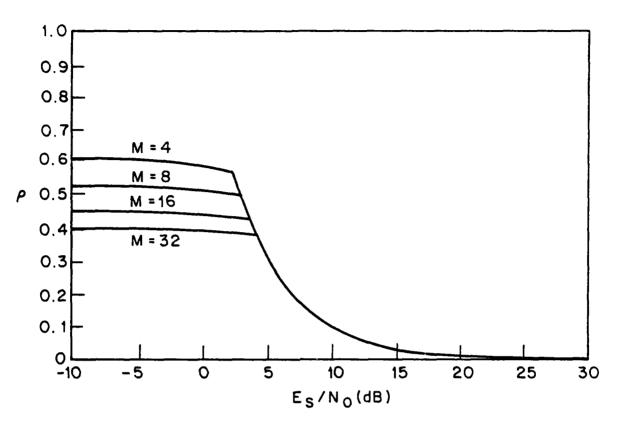


FIG. 7 - Worst Case Jamming Fraction for List-of-M Detection on a Multiple-Tone Channel

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